6.832 Midterm

Name: ______________________________

November 3, 2015

Please do not open the test packet until you are asked to do so.

- You will be given 85 minutes to complete the exam.
- Please write your name on this page, and on any additional pages that are in danger of getting separated.
- We have left workspace in this booklet. Scrap paper is available from the staff. Any scrap paper should be handed in with your exam.
- YOU MUST WRITE ALL OF YOUR ANSWERS IN THIS BOOKLET (not the scrap paper).
- The test is open notes.
- The test is out of 37 points. (The 4th problem is omitted from this version.)

Good luck!

Problem 1 (10 pts) *Lyapunov analysis*

Consider the system described by:

$$\dot{x} = x - x^3$$

In this problem we will investigate the stability of the fixed point at $x^* = 1$.

a) *Draw the function* $\dot{x}$ *as a function of* x *in the space below. Circle the fixed points, and label each fixed point as stable or unstable.*

b) *Linearize the system about the fixed point at* $x^* = 1$. *Write your answer in the form:* $\dot{\bar{x}} = A\bar{x}$, *where* $\bar{x} = x - x^*$.

c) *Solve the Lyapunov equation* $(PA + A^T P = -Q, P = P^T \succ 0)$ *using* $Q = 1$ *to find a candidate Lyapunov function. Write the resulting Lyapunov function as a polynomial in terms of* x*:*

d) *Compute* $\dot{V}$ *using the original nonlinear dynamics.*

e) What initial conditions x does this Lyapunov candidate prove are inside the region of attraction of the fixed point? (Hint: all roots of the resulting polynomial are at integer values)

f) Is this estimate of the region of attraction tight? (If no, then demonstrate this by naming another initial condition in the region of attraction which is outside the certified region).

g) *With the Lyapunov candidate, $V(\mathbf{x})$ given, is it possible to certify this region of attraction using a single **convex** sums-of-squares optimization?*

YES or NO

- *If yes, what are the decision variables (you need not provide the decision variables required to write the SOS program as an SDP)?*
- *If no, then explain why the problem is not convex.*

[You may wish to write down the optimization to make your answer clear]

This page intentionally left blank for workspace

Problem 2 (10 pts) *Planar quadrotors*

Consider a planar quadrotor model with two ideal thrusters and a simple model of bluff-body drag and all non-gravity constants set to 1*, given by*

$$\ddot{x} = -\sin\theta(u_1 + u_2) - \dot{x}^2$$
$$\ddot{z} = -g + \cos\theta(u_1 + u_2) - \dot{z}^2$$
$$\ddot{\theta} = -u_1 + u_2.$$

This model has two inputs to control three degrees of freedom; intuitively we should be able to control two of them with feedback linearization (but note that the coupling in the first two equations prevents us from commanding an arbitrary $\ddot{x}_d, \ddot{z}_d$*). Assume* u *is unbounded unless otherwise noted.*

a) *Give a partial-feedback linearizing controller that imposes the closed-loop dynamics*

$$\ddot{\theta} = \ddot{\theta}_d, \quad \ddot{z} = \ddot{z}_d.$$

b) Does your feedback controller have any singularities? If so, in what states?

Now imagine that you've lost a propellor ($u_2 = 0$). You're quadrotor is going down. Fortunately, you have been given the optimal cost-to-go function, $J^*(\mathbf{x})$, for a damage-minimizing cost function of the form

$$J = \int_0^\infty g(\mathbf{x}) dt.$$

This time you have to consider your input limits, $0 \leq u_1 \leq u_{max}$.

c) What is the optimal policy, $u_1 = \pi^*(\mathbf{x})$ as a function of g, J^*, and its partial derivatives?

This page intentionally left blank for workspace

Problem 3 (7 pts) *Scaling up*

As a part of your latest robotic art installation, you've assembled a robot with 284 degrees of freedom, and you want it to be dynamic, but you could only afford 35 actuators. Which of the following techniques from class might you have a hope of applying to your contraption? Justify your answers. (Note: the problem is subjective; your job is to write something for each method to convince us you understand)

a) *Value iteration using barycentric interpolation*

b) *Linear Quadratic Regulator (LQR), using a linearization of the system in the vicinity of a fixed point*

c) *Partial feedback linearization*

d) Lyapunov's method for linear systems $(PA + A^TP = -Q, P = P^T \succ 0)$

e) Sums-of-squares optimization for estimating regions of attraction

f) Sums-of-squares optimization for controller design *g) Nonlinear trajectory optimization using direct collocation and sequential quadratic programming*

h) Linear trajectory optimization using quadratic programming, using a linearization of the system in the vicinity of a fixed point

This page intentionally left blank for workspace